MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2016/2017

PCL0016 – CALCULUS

(FOUNDATION ENGINEERING)

25 February 2017
2.30p.m. – 4.30p.m.
(2 Hours)

INSTRUCTION TO STUDENT

1. This examination paper consists of **4 pages** (including the cover page) with **6 questions only**.

2. Questions **1** and **6** are allocated with **20** marks while Questions **2**, **3**, **4** and **5** are allocated with **15** marks. **Attempt ALL questions.**

3. Please write all your answers in the Answer Booklet provided. **Show all relevant steps** to obtain maximum marks.

Question 1 [20 marks]

(a) A piecewise function $f(x)$ is denoted as

$$f(x)=\begin{cases}1, & x<0\\ 4, & x=0\\ 2x^2+1, & 0<x<1\\ x, & x\geq 1\end{cases}.$$

(i) Find $\lim_{x\to -1} f(x)$ and $\lim_{x\to 1} f(x)$. [2 marks]

(ii) Show that $f(x)$ is not continuous at $x = 0$. [3 marks]

(b) Evaluate the following limits, if they exist

(i) $\lim_{x\to 9}\dfrac{\sqrt{x}-3}{x-9}$ [2 marks]

(ii) $\lim_{x\to 2}\dfrac{3x-6}{2x^2-8}$ [2 marks]

(iii) $\lim_{x\to -\infty}\dfrac{2x^{20}+5x^{10}+1}{7x^{21}+6x^{11}+11x^2}$ [2 marks]

(iv) $\lim_{x\to 0}\dfrac{\sin 3x}{\tan 2x}$ [3 marks]

(c) (i) Using Squeeze Theorem, show that $\lim_{x\to\infty}\dfrac{\cos x}{x+3}=0$. [3 marks]

(ii) Evaluate $\lim_{x\to\infty}\dfrac{2x^4+2}{e^{2x}}$ using L'Hôpital's rule. [3 marks]

Question 2 [15 marks]

(a) Find $\dfrac{dy}{dx}$ for the following functions.

(i) $y=\ln(3x)^x$ [3 marks]

(ii) $y=\sin^2 x^2$ [3 marks]

(iii) $x^3+\cos y=3y^2$ [3 marks]

Continued...

(b) (i) Determine an expression for the slope of a tangent line to the curve $y=\dfrac{x^2+2}{x+1}$ [2 marks]

(ii) At which points on the given curve are the tangent lines (found in (b)(i)) perpendicular to $y=-4x+1$? [4 marks]

Question 3 [15 marks]

A function $f(x)$ is denoted by $f(x)=-x^2(x-1)(x+1)$.

(a) Find the x-intercept(s) and y- intercept(s) of the graph of f, if any. [2 marks]

(b) Find the interval(s) on which $f(x)$ is increasing and/or decreasing. [4 marks]

(c) Find the interval(s) on which $f(x)$ is concave up and/or down. [3 marks]

(d) Identify the relative extrema of $f(x)$. [3 marks]

(e) Sketch the graph of $f(x)$. [3 marks]

Question 4 [15 marks]

(a) Using integration by parts, show that

$$\int_0^{\frac{\pi}{6}} x\sin 2x\,dx=\frac{\sqrt{3}}{8}-\frac{\pi}{24}.$$ [5 marks]

(b) Given $f(x)=\dfrac{9x^3-3x+1}{x^3-x^2}$. Evaluate $\int f(x)dx$ using long division and partial fractions. [7 marks]

(c) Evaluate the following integral using integration by substitution.

$$\int\frac{4x}{\sqrt{2x^2+1}}dx$$ [3 marks]

Continued...

Question 5 [15 marks]

A region R is bounded by the curve $y = e^x$ and the lines $x = 1$, $x = 3$ and $y = 25$.

(a) Sketch and shade the region R. [5 marks]

(b) Find the area of the region R, rounding off your answer to 2 decimal places. [4 marks]

(c) Using the washer method, find the volume of the solid generated when the region R is revolved around the x-axis. Round off your answer to 2 decimal places. [6 marks]

Question 6 [20 marks]

For the following differential equations, find particular solutions expressed explicitly.

(a) $\frac{dy}{dx} = \sqrt{xy^3}$, $y(0) = 4$. [5 marks]

(b) $y'' + 2y' + y = 0$, $y(0) = 1$, $y'(0) = 1$. [7 marks]

(c) $\frac{dy}{dx} = \frac{xy - y^2}{x^2}$, $y(1) = 1$. Hint: Use $y = vx$) [8 marks]

End of Paper